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A Two-parameter Family fo Socio-economic Health Inequality Indices:

Accounting for Risk and Inequality Aversions

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A Two-parameter Family of Socio-economic Health Inequality Indices: Accounting for Risk and Inequality Aversions*

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Abstract

This paper proposes a two-parameter family of socio-economic health inequality indices. First, these indices allow a Boolean risk factor to be linked to other health dimensions. Second, multidimensional health distributions can be compared thanks to a stochastic dominance rule, which includes the attitude of the social planner with respect to the risk factor (risk neutrality, risk aversion and extreme risk aversion). Third, each order of stochastic dominance is also associated with the intensity of possible health transfers occurring between individuals, that is, the degree of inequality aversion of the social planner. This approach is a multidimensional extension of Yitzhaki's Gini indices accounting simultaneously for risk and redistribution.

Key words: Counting, Dominance, Health, Inequality, Risk.

Classification JEL: D6, I1.

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1 Introduction

The measurement of socio-economic health inequalities has mainly been studied according to the well-known concentration indices. These concentration indices were first handled because of their simple interpretations and also for their econometric properties – see for instance Kakwani, Wagstaff and van Doorslaer (1997). They are also commonly employed since, following Yitzhaki (1983), concentration indices are relevant for a given degree of inequality aversion supported by the decision maker.

In the case of socio-economic health inequality indices, a first line of research demonstrates the necessity of employing several decomposable indices. In the framework of unidimensional indicators of health, Wagstaff, van Doorslaer and Watanabe (2003) show that socio-economic health inequality indices are decomposable into the contribution of different explicative factors such as the levels of education and consumption, amongst others. Recently, this property, called attribute decomposition, has been extended to the multidimensional context. Makdissi, Sylla and Yazbeck (2013), based on the rank-dependent approach, show that this decomposition property is matched when the health dimensions – which are categorical dimensions – are defined to be Boolean variables. This counting approach allows each dimension to be gauged in proportion to the overall amount of the socio-economic health inequality index and, at the same time, it involves a parameter for the intensity of health redistribution to be done. The idea of capturing the role of one or many dimensions will be of interest in what follows.

A second line of research has been devoted to the notion of risk. On the one hand, risk is associated with genes at birth as well as choices, also called risky behaviors, *e.g.* addictive habits, see Gakidou, Murray and Frenk (1999). In this line of research, Le Clainche and Wittwer (2015) prove that students are inclined to support the health costs following from their risky choices. On the other hand, the notion of risk is analyzed in the context of equality of opportunity, as developed by Dworkin (1981), Arneson (1989), Cohen (1989), Roemer (1998), and Fleurbaey (2008). In particular, several authors try to distinguish between the legitimate and the illegitimate causes of health inequalities, name separating the efforts and lifestyle associated with the notion of risky behaviors from the circumstances which are beyond an individual's control, associated with the notion of exogenous risks. For example, Trannoy, Tubeuf, Jusot and Devaux (2010), investigate inequality of opportunity in health by analyzing the role of circumstances during childhood, such as family and social background. In the remainder of the paper, we will study the risk factors that are not under an individual's control –

exogenous factors.

In this paper, the two aforementioned lines of research are combined. The aim is to provide a multidimensional socio-economic health inequality index, which depends on different health dimensions and also on one exogenous risk factor. The idea is to conceive an index that will reflect two very different attitudes of the decision-maker behind the veil of ignorance. The first one is the usual attitude towards inequality as embodied by the degree of aversion to inequality (see Yitzhaki, 1983), *i.e.* the willingness of the social planner to operate redistributive policies toward less healthy people in order to alleviate overall inequality in a society. The second one is the degree of risk aversion of the social planner. The literature advocates, amongst others, the use of Yaari's (1987) dual social welfare function in order to involve different degrees of risk aversion in the analysis. In our approach, the degree of risk aversion is derived from the association between the health dimensions and the risk factor (considered as an additional dimension). Also, the risk aversion parameter allows the health dimensions to be properly aggregated, while the aggregation process is sensitive to the Boolean values inherent in the risk dimension.

The advantage of dealing with the proposed socio-economic health inequality indices is threefold. First, it allows a Boolean risk factor to be associated with other health dimensions (*e.g.* physical and mental ones). Second, the comparison of multidimensional health distributions relies on a simple graphical approach. Indeed, a stochastic dominance criterion provides a non-ambiguous ranking of health distributions including one risk factor. In this respect, the dominance rule is compatible with either risk neutrality, risk aversion, or extreme risk aversion. As a consequence, the social planner's attitudes to risk are captured for each order of stochastic dominance. Third, each order of stochastic dominance is properly associated to redistributive health actions (transfers) *i.e.* to the degree of inequality aversion of the social planner. Hence, the dominance rule depends simultaneously on two parameters, one parameter that represents risk aversion, and another parameter that embodies inequality aversion. Finally, this approach is a multidimensional extension of Yitzhaki's Gini indices (1983), for which it is possible to calibrate both inequality and risk aversions. This provides a two-parameter family of socio-economic health inequality indices.

On the basis of 1,610 Luxembourgish households, we apply our stochastic dominance rule by changing the exogenous risk factors and by comparing all possible multidimensional health distributions. We find, among a wide range of Boolean risk factors concerned with the parents' characteristics, that the most important exogenous risk factors explaining health inequality in

Luxembourg are the nationality as well as the education level of the parents. This result is robust for risk averse as well as extreme risk averse decision makers, insofar it is concerned, they are inclined to perform health transfers.

The paper is organized as follows. In Section 2 the family of rank-dependent socio-economic health inequality indices is set out. In Section 3, the risk properties are presented and they are linked with risk neutrality and/or risk aversion. The stochastic dominance criterion associated with the family of two-parameter socio-economic health inequality indices is proposed in Section 4. An application is performed on Luxembourgish households in Section 5. Section 6 presents some robustness checks. Section 7 concludes the paper.

2 Health Inequality and Health Achievement

This section briefly summarizes the notations and definitions used in the paper.

2.1 Notations

Let y^E be an equivalent income distribution such that $F\left(y^E\right)$ is its cumulative distribution function defined over $[0, a]$, where a is the maximum conceivable equivalent income. There are n individuals in the society, where n is a positive integer. The rank p of the individuals are issued from $F\left(y^E\right)$, such that $p \in [0, 1]$. Following the literature on the counting approach for measuring poverty, see *e.g.* Alkire and Foster (2011), we adopt the counting approach to gauge inequalities in multidimensional health in line with Makdissi and Yazbeck's (2014) approach. Let $\mathbf{H}(p) := (h_1(p), \ldots, h_K(p)) \in \mathbb{R}_+^K$ be the information related to each dimension of health indexed by $k \in \{1, \ldots, K\} =: \mathcal{K}$ for an individual at rank p of the equivalent income distribution, and where $\mathbb{R}_+^K$ is the K-dimensional Euclidean space such that K is a (strictly) positive integer.

The set of all possible health information (matrices) is denoted $\mathcal{H}$. The $n \times K$ health information matrix is $\mathbf{H} \in \mathcal{H}$ such that $\mathbf{H} = (h_1, \ldots, h_K)$ where h_k denotes the kth column of $\mathbf{H}$, whereas $\mathbf{H}(p)$ denotes a row of $\mathbf{H}$ for an individual at rank p. $\mathbf{H}_{-p}$ denotes the health information matrix $\mathbf{H}$ without the row $\mathbf{H}(p)$ and $[\mathbf{0}]$ a matrix of zeros. This health information is derived from an identification function *i.e.* a threshold function τ_k. The threshold τ_k, for each dimension k, indicates for any given individual at rank p whether he falls below the threshold τ_k, in which case the individual is considered 'deprived' in the health dimension k. Thus, the individual is counted 1 in dimension k, otherwise 0, which indicates that there is no 'deprivation' in the health dimension k:

$$\iota(h_k(p)) := \begin{cases} 1, & \text{if } h_k(p) < \tau_k \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

Accordingly,

$$\Upsilon(\mathbf{H}(p)) := \big(\iota(h_1(p)), \dots, \iota(h_k(p)), \dots, \iota(h_K(p))\big), \tag{2.2}$$

provides the K-dimensional situation of an individual at rank p of the equivalent income distribution, and $\Upsilon(\mathbf{H})$ a $n \times K$ Boolean matrix.

2.2 Definitions

In what follows, we define a normalized aggregator, a map ϕ, in order to aggregate their health dimensions for an individual at rank p. In welfare economics, the usual aggregator is the generalized mean introduced and axiomatized by Blackorby, Donaldson and Auersperg (1981).

Definition 2.1 – **Aggregator** – *Let* $\|\cdot\|_\alpha^\Theta : \mathbb{R}_+^K \longrightarrow \mathbb{R}_+$ *be a twice differentiable map such that for all* $x \in \mathbb{R}_+^K$ *and some weight vector* $\Theta := (\theta_1, \dots, \theta_K) \in [0,1]^K$*:*

$$\|x\|_\alpha^\Theta := \begin{cases} \left(\sum_{k=1}^K \theta_k x_k^\alpha\right)^{\frac{1}{\alpha}} & \forall \alpha > 0 \\ \prod_{k=1,\dots,K} x_k^{\theta_k} & \alpha \to 0 \text{ (or } \alpha = 0 \text{ by convention).} \end{cases} \tag{2.3}$$

A normalized aggregator function $\phi : [0,1]^K \longrightarrow [0,1]$ is:[1]

$$\phi(\mathbf{H}(p)) = \frac{\|\Theta\|_\alpha^{\mathbf{1}_K} - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta}{\|\Theta\|_\alpha^{\mathbf{1}_K}}, \tag{2.4}$$

where $\mathbf{1}_K$ denotes the K-dimensional vector of ones. Without loss of generality, one may impose that $\sum_k \theta_k = 1$, and so, ***

$$\|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta = \left(\sum_{k=1}^K \theta_k \ \iota(h_k(p))^\alpha\right)^{\frac{1}{\alpha}} \leq \max_{k=1,\dots,K} \iota(h_k(p)). \tag{2.5}$$

[1]For instance Makdissi and Yazbeck (2014) propose:

$$\phi(\mathbf{H}(p)) = \frac{K - \Upsilon(\mathbf{H}(p))\Theta'}{K}, \text{ where } \sum_{k=1}^K |\theta_k| = K.$$

Accordingly,

$$\|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta} \leq \|\Theta\|_{\alpha}^{1_K} \leq \max_{k=1,\dots,K} \theta_k = 1. \tag{2.6}$$

In this case, the normalized aggregator may be simply expressed as follows:

$$\phi_{\alpha}(\mathbf{H}(p)) := 1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta}. \tag{2.7}$$

The aggregator $\phi_{\alpha}(\mathbf{H}(p))$ represents the average health *achievement* for an individual at rank p. The overall socio-economic health achievement index is then,

$$A_{\alpha}(\mathbf{H}) := \int_0^1 v(p)\phi_{\alpha}(\mathbf{H}(p))dp = \int_0^1 v(p)\left(1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta}\right)dp, \tag{2.8}$$

where $v(p)$ is a rank-dependent weight function such that $v : [0,1] \longrightarrow [0,1]$ which embodies the social planner's preferences.[2] The index $A_{\alpha}(\mathbf{H})$ is a natural K-dimensional extension of the concentration index analyzed for instance by Wagstaff *et al.* (2003) or Erreygers, Clarke and Van Ourti (2012), amongst others. It represents a weighted mean of the *individual achievements* $1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta}$ across all individuals in a society. In the multidimensional health literature, Makdissi *et al.* (2013) suggest the use of $v(p) = \nu(1-p)^{\nu-1}$ in line with Yitzhaki's (1983) extended Gini indices. In this case, a two-parameter family of achievement indices can be obtained as follows:

$$A_{\nu,\alpha}(\mathbf{H}) := \int_0^1 \nu(1-p)^{\nu-1}\phi_{\alpha}(\mathbf{H}(p))dp, \ \nu > 1, \ \alpha \geq 0. \tag{2.9}$$

If $\nu \geq 2$ the index displays health inequality aversion, whereas health inequality loving is obtained whenever $\nu \in (1,2)$. If $\nu = 2$ the well-known concentration index is deduced.

Definition 2.2 – Socio-economic health inequality indices – *For all* $\mathbf{H} \in \mathcal{H}$ *and* $\mu_{\phi(\mathbf{H})} = \int_0^1 \phi_{\alpha}(\mathbf{H}(p))dp$, *the two-parameter family of socio-economic health inequality indices is given by:*

$$\begin{aligned} I_{\nu,\alpha}(\mathbf{H}) &= 1 - \frac{A_{\nu,\alpha}(\mathbf{H})}{\mu_{\phi(\mathbf{H})}} \qquad (2.10) \\ &= 1 - \frac{1}{\mu_{\phi(\mathbf{H})}} \int_0^1 \nu(1-p)^{\nu-1}\left(1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta}\right)dp, \ \nu > 1, \ \alpha \geq 0. \end{aligned}$$

In what follows, the α parameter will be linked to the risk aversion of the social planner, which is derived from the association between the risk dimension and the health dimensions. The advantage of dealing with a two-parameter family of socio-economic health inequality measures, denoted in a generic manner from now on as $\{I\}_{\nu,\alpha}$, is the possibility of capturing risk sensitive properties.

[2] See the seminal work of Yaari (1987).

3 Index parametrization with risk sensitive properties

In this section, emphasis is put on the notion of risk. The different subsections describe the properties of $\{I\}_{\nu,\alpha}$ with respect to one risk factor. The last subsection explores another type of parametrization, one related to distributional purposes.

3.1 Boolean risk factor

We consider that within health information, one dimension represents a risk factor that may cause health failure. In the remainder, for ease of exposition, when comparing two health matrices $\mathbf{H}, \tilde{\mathbf{H}} \in \mathcal{H}$ we suppose without loss of generality that $\mu_{\phi(\mathbf{H})} = \mu_{\phi(\tilde{\mathbf{H}})}$.[3]

Definition 3.1 – One-dimensional risk factor – *For all distributions $\mathbf{H} \in \mathcal{H}$, the risk factor potentially correlated to the health dimensions is defined to be dimension K, by convention, such that $\iota(h_K) \equiv \iota(h_{\mathbf{r}}) \in \{0,1\}$, with $\iota(h_{\mathbf{r}}(p)) = 0$ for an individual at rank p not affected by a risk factor and 1 otherwise. The set of health information is accordingly decomposed such that $\mathcal{K} = \{1, \ldots, K-1\} \cup \{\mathbf{r}\} \equiv \mathcal{K}_{-\mathbf{r}} \cup \{\mathbf{r}\}$, with $\mathbf{H}_{-\mathbf{r}} := (h_1, \ldots, h_{k-1})$, so that $\mathbf{H} = (\mathbf{H}_{-\mathbf{r}}, h_{\mathbf{r}})$.*

Although $\iota(h_{\mathbf{r}})$ could be a continuous function of many risk factors, we suppose without loss of generality that only one risk dimension is available. An option would be to consider that $\iota(h_{\mathbf{r}})$ itself depends on r Boolean risk factors such that $r = 1, \ldots, R$ with $\iota(h_{\mathbf{r}}) = \|(\iota(h_{\mathbf{r},1}), \ldots, \iota(h_{\mathbf{r},R}))\|_{\alpha_{\mathbf{r}}}^{\Theta_{\mathbf{r}}}$, and where $\alpha_{\mathbf{r}}$ and $\Theta_{\mathbf{r}}$ are weights that are risk specific. Those different strategies of averaging risk factors will be discussed in the empirical section of the paper. For the properties developed below, it is just necessary to get a bounded risk factor included in [0,1]. For simplicity, it is assumed that $\iota(h_{\mathbf{r}}) \in \{0,1\}$.

3.2 Risk neutrality and risk aversion

The first sensibility property is related to the usual union/intersection approach of the literature on multidimensional poverty, introduced by Atkinson (2003). The union approach is the less demanding value judgment of the social planner in charge of the identification of healthy people across dimensions. It postulates that an individual has to be non healthy in all dimensions to be considered as non healthy. In other terms, an individual is considered

[3] Indeed, we will show in Section 4 that, for dominance purposes, no condition has to be imposed on the mean.

as totally non healthy when $\phi_\alpha(\mathbf{H}(p)) = 1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta = 0$. In contrast, an individual is totally healthy if $\phi_\alpha(\mathbf{H}(p)) = 1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta = 1$. Suppose that an individual is healthy in at least one dimension, then there exists one dimension $k \in \mathcal{K}_{-\mathbf{r}}$ such that $\iota(h_k(p)) = 0$. In this respect, such an individual is judged to be totally healthy in the same manner as an individual who is healthy in all dimensions. In particular, if an individual is affected by a risk factor *i.e.* $\iota(h_\mathbf{r}(p)) = 1$ then this dimension would actually be neutral. Formally, whenever $\alpha \to 0$, the risk neutrality implies that the individual is healthy if it is the case that $\iota(h_\mathbf{r}(p)) = 1$:

$$\phi_0(\mathbf{H}(p)) := \lim_{\alpha \longrightarrow 0} \left(1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta\right) = 1 - \prod_{k=1}^{K} \iota(h_k(p))^{\theta_k} = 1. \tag{3.1}$$

The opposite property, the more demanding one, is known to be the intersection approach. Suppose that an individual is only deprived in the risk dimension. The decision maker is going to judge this person as totally non healthy, in the same manner as a person who is non healthy in all dimensions. In this case, the risk aversion is maximal since, for any given health state $\mathbf{H}_{-\mathbf{r}}(p)$ of an individual ranked p, the risk factor $\iota(h_\mathbf{r}(p)) = 1$ will provide a non-healthy state. Formally, whenever $\alpha \to \infty$ (or $\alpha = \infty$ by convention):

$$\phi_\infty(\mathbf{H}(p)) := \lim_{\alpha \longrightarrow \infty} \left(1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta\right) = 1 - \max_{k=1,\dots,K} \iota(h_k(p)) = 0. \tag{3.2}$$

As a consequence, the α parameter enables to capture the preference of the social planner with respect to the health dimensions associated with one risk factor.[4]

Property 3.1 – Risk neutrality / Risk aversion – $\mathcal{RN}/\mathcal{RA}$ **:**
The aggregator $\phi_\alpha(\mathbf{H}(p)) = 1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta$ *yields either risk neutrality or different degrees of risk aversion, which are characterized by the following sets of health distributions.*
(i) *Risk Neutrality:*

$$\mathcal{RN} := \left\{ \mathbf{H}, \widetilde{\mathbf{H}} \in \mathcal{H} \;\middle|\; \begin{array}{l} \widetilde{\mathbf{H}} \textit{ is issued from } \mathbf{H} \textit{ such that} \\ \exists k \in \mathcal{K}_{-\mathbf{r}} : \iota(h_k(p)) = \iota(\tilde{h}_k(p)) = 0, \\ \iota(h_\mathbf{r}(p)) = 1, \iota(\tilde{h}_\mathbf{r}(p)) = 0, \\ \phi_0(\mathbf{H}(p)) = \phi_0(\widetilde{\mathbf{H}}(p)). \end{array} \right\}$$

(ii) *Risk Aversion:*

[4] It is noteworthy that in the literature on multidimensional poverty the union and intersection approaches rely on the number of dimensions to be considered as deprived or not, rather than a parameter directly linked to the aggregation of the dimensions (see Alkire and Foster, 2011).

$$\mathcal{RA} := \left\{ \mathbf{H}, \widetilde{\mathbf{H}} \in \mathcal{H} \;\middle|\; \begin{array}{l} \widetilde{\mathbf{H}} \textit{ is issued from } \mathbf{H} \textit{ such that } \mathbf{H}_{-\mathbf{r}} = \widetilde{\mathbf{H}}_{-\mathbf{r}}, \\ \iota(h_{\mathbf{r}}(p)) = 1, \iota(\tilde{h}_{\mathbf{r}}(p)) = 0, \\ \phi_\alpha(\mathbf{H}(p)) < \phi_\alpha(\widetilde{\mathbf{H}}(p)), \alpha \in (0, \infty). \end{array} \right\}$$

(iii) *Extreme Risk Aversion:*

$$\mathcal{ERA} := \left\{ \mathbf{H}, \widetilde{\mathbf{H}} \in \mathcal{H} \;\middle|\; \begin{array}{l} \widetilde{\mathbf{H}} \textit{ is issued from } \mathbf{H} \textit{ such that} \\ \Upsilon(\mathbf{H}_{-\mathbf{r}}) = \Upsilon(\widetilde{\mathbf{H}}_{-\mathbf{r}}) = [\mathbf{0}], \\ \iota(h_{\mathbf{r}}(p)) = 1, \iota(\tilde{h}_{\mathbf{r}}(p)) = 0, \\ \phi_\infty(\mathbf{H}(p)) = 0 < 1 = \phi_\infty(\widetilde{\mathbf{H}}(p)). \end{array} \right\}$$

The $\mathcal{RN}$ property displays the risk neutrality of the decision-maker. When the individuals are healthy in one dimension only, for any given risk level (0 or 1), the individual is considered healthy. On the other hand, to be considered non-healthy, the individuals must be non-healthy in all dimensions without any distinction between the risk factor and the other dimensions. The risk aversion property $\mathcal{RA}$ directly depends on the risk factor, which, *ceteris paribus*, decreases the individual health achievement.The extreme risk aversion view $\mathcal{ERA}$ is the intersection approach explained above. A non-healthy state is assigned to an individual exposed to a risk factor even if he is totally well-off in all health dimensions $\mathcal{K}_{-\mathbf{r}}$.

As can be seen in Property 3.1, a clear separation is made between the health dimensions and the risk factor. It is noteworthy that this possibility is inherent to the generalized mean aggregator $\|\cdot\|_\alpha^\Theta$, which is characterized by the additive separability axiom – see Blackorby *et al.* (1981). However, this property is not incompatible with the design of the correlations between the risk factor and the other dimensions.

3.3 Correlation increasing risk

This degree of correlation, in a multidimensional framework, has to be studied through the prism of *correlation increasing risk*, introduced by Richard (1975) and formerly by Meyer (1972) in the unidimensional setting. This concept has been suitably used for multivariate measurement tools, known as correlation increasing switch, see *e.g.* Atkinson and Bourguignon (1982) and under other names by Boland and Proschan (1988), Tsui (1999), and Seth (2013), amongst others. The idea underlying those concepts is to capture the interaction between the dimensions, their complementarity as well as their substitutability, in the same manner as the sensitivity towards risk discussed above. In particular, the parameter α may be connected to the degree of correlation between health dimensions.

Consider two persons, say for simplicity p_1 and p_2, to denote the individuals at rank p_1 and p_2, respectively. Suppose that p_1 receives the maximum achievement amount (denoted $\vee$) among all dimensions $k \in \mathcal{K}$, including the risky one, between his situation and that of p_2, and that in contrast, p_2 receives the minimum between their achievements (denoted $\wedge$) across all dimensions. Actually, this *correlation increasing risk* aggravates the inequality between p_1 and p_2 while each individual situation appears to be smoother (all 0's for p_2 and all 1's for p_1). The inequality is more important because one individual would be non healthy in all dimensions and consequently would support the entire risk factor. If $\widetilde{\mathbf{H}}$ denotes the health information after a correlation increasing risk, then:

$$\left\|\Upsilon(\widetilde{\mathbf{H}}(p_1))\right\|_\alpha^\Theta := \|\Upsilon(\mathbf{H}(p_1)) \vee \Upsilon(\mathbf{H}(p_2))\|_\alpha^\Theta \tag{3.3}$$

$$\left\|\Upsilon(\widetilde{\mathbf{H}}(p_2))\right\|_\alpha^\Theta := \|\Upsilon(\mathbf{H}(p_1)) \wedge \Upsilon(\mathbf{H}(p_2))\|_\alpha^\Theta , \tag{3.4}$$

that is,

$$\left\|\Upsilon(\widetilde{\mathbf{H}}(p_1))\right\|_\alpha^\Theta = \|\iota(h_1(p_1)) \vee \iota(h_1(p_2)), \cdots , \iota(h_\mathbf{r}(p_1)) \vee \iota(h_\mathbf{r}(p_2))\|_\alpha^\Theta \tag{3.5}$$

$$\left\|\Upsilon(\widetilde{\mathbf{H}}(p_2))\right\|_\alpha^\Theta = \|\iota(h_1(p_1)) \wedge \iota(h_1(p_2)), \cdots , \iota(h_\mathbf{r}(p_1)) \wedge \iota(h_\mathbf{r}(p_2))\|_\alpha^\Theta . \tag{3.6}$$

The concept of correlation increasing risk relies on the idea that the decision maker would have the possibility of switching the risk level to which some individuals are exposed. In what follows, the set $\mathcal{CIR}$ denotes the set of all distributions $\mathbf{H}, \widetilde{\mathbf{H}} \in \mathcal{H}$ for which a correlation increasing risk is applied such that $\Upsilon(\mathbf{H}_{-\mathbf{r}}(p)) = \Upsilon(\widetilde{\mathbf{H}}_{-\mathbf{r}}(p)) \neq [\mathbf{0}]$ for all $p \in [0, 1]$ and $\iota(h_\mathbf{r}(p_1)) \neq \iota(h_\mathbf{r}(p_2))$ – with $\mathcal{CIR}$ being used indifferently as a set or a property label. In other terms, a pure permutation is made between the individuals ranked p_1 and p_2. Also, the sequence of correlation increasing risk must be non void, which could occur if $\mathbf{H}(p_1) = \mathbf{H}(p_2)$ since in this case $\mathbf{H}(p_1) = \widetilde{\mathbf{H}}(p_1)$ and $\mathbf{H}(p_2) = \widetilde{\mathbf{H}}(p_2)$.

Property 3.2 – Correlation Increasing Risk – $\mathcal{CIR}$ **:**
For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, if $(\widetilde{\mathbf{H}}, \mathbf{H}) \in \mathcal{CIR}$ such that $\widetilde{\mathbf{H}}$ is issued from $\mathbf{H}$ by a non-void correlation increasing risk between two individuals, then:

$$I_{\nu,\alpha}(\widetilde{\mathbf{H}}) > I_{\nu,\alpha}(\mathbf{H}).$$

On this basis, it is possible to restrict the values of the α parameter thanks to $\mathcal{CIR}$.

Lemma 3.1 *For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, if a non void sequence of correlation increasing risk between two individuals p_i and p_j is applied such that $p_i = p_j$, then the two following statements hold.*
(i) *Assume that the aggregator $\|\cdot\|_{\alpha}^{\Theta}$ is approximated by a twice differentiable function $f : [0,1]^K \longrightarrow [0,1]$. Then, $I_{\nu,\alpha}$ respects $\mathcal{CIR}$ if and only if $\|\cdot\|_{\alpha}^{\Theta}$ is strictly L-superadditive and $\alpha > 1$.*
(ii) *$I_{\nu,\alpha}$ is invariant to any $\mathcal{CIR}$ if and only if $\alpha \in \{0, 1, \infty\}$. Then,*
(ii.a) *$\mathcal{CIR} \cap \mathcal{RN} = \emptyset$.*
(ii.b) *$\mathcal{CIR} \cap \mathcal{RA} = \emptyset$ whenever $\alpha = 1$.*
(ii.c) *$\mathcal{CIR} \cap \mathcal{ERA} = \emptyset$.*

Proof:
(i) Let $I_{\nu,\alpha}(\widetilde{\mathbf{H}}) > I_{\nu,\alpha}(\mathbf{H})$ or equivalently $A_{\nu,\alpha}(\widetilde{\mathbf{H}}) < A_{\nu,\alpha}(\mathbf{H})$. Taking two individuals ranked p_i and p_j, if their rank after a $\mathcal{CIR}$ are denoted p_i' and p_j' respectively, then:

$$\begin{aligned} & v(p_i')\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_i))\|_{\alpha}^{\Theta}\right) + v(p_j')\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_j))\|_{\alpha}^{\Theta}\right) \\ < \; & v(p_i)\left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_{\alpha}^{\Theta}\right) + v(p_j)\left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\Theta}\right). \end{aligned} \tag{3.7}$$

Since $v(p) \geq 0$ for all $p \in [0,1]$ and since the equivalent income y^E is by definition invariant to any $\mathcal{CIR}$, then $p_i = p_j = p_i' = p_j'$, and so:

$$\begin{aligned} & \|\Upsilon(\mathbf{H}(p_i)) \vee \Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\Theta} + \|\Upsilon(\mathbf{H}(p_i)) \wedge \Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\Theta} \\ > \; & \|\Upsilon(\mathbf{H}(p_i))\|_{\alpha}^{\Theta} + \|\Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\Theta}. \end{aligned} \tag{3.8}$$

Let $f(x) = \|x\|_{\alpha}^{\Theta}$ such that $f : [0,1]^K \longrightarrow [0,1]$ is twice differentiable. From Eq.(3.8) it follows that f is strictly L-superadditive, see Boland and Proschan (1988) and Tsui (1999). A function $f : [0,1]^K \longrightarrow [0,1]$ is L-superadditive if, and only if, $\frac{\partial^2 f(x_1,\ldots,x_K)}{\partial x_k \partial x_j} \geq 0$. After simple algebraic manipulations, it can be shown that strict L-superadditivity is ensured whenever $\alpha > 1$.
(ii) Consider a correlation increasing risk between two individuals p_i and p_j such that $p_i = p_j$. Since $v(p_i) = v(p_j) = v(p_i') = v(p_j')$, then from (3.8):

$$\begin{aligned} & \left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_i))\|_{\alpha}^{\Theta}\right) + \left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_j))\|_{\alpha}^{\Theta}\right) \\ < \; & \left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_{\alpha}^{\Theta}\right) + \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\Theta}\right). \end{aligned} \tag{3.9}$$

(ii.a) From $\mathcal{RN}$, there exists $k \in \mathcal{K}_{-\mathbf{r}}$ such that $\iota(h_k(p)) = 0$, then a contradiction arises:

$$\begin{aligned} & \lim_{\alpha \to 0}\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_i))\|_{\alpha}^{\Theta}\right) + \lim_{\alpha \to 0}\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_j))\|_{\alpha}^{\Theta}\right) \\ = \; & \lim_{\alpha \to 0}\left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_{\alpha}^{\Theta}\right) + \lim_{\alpha \to 0}\left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\Theta}\right). \end{aligned} \tag{3.10}$$

(ii.b) Setting $\alpha = 1$ in the $\mathcal{RA}$ case yields also a contradiction since,

$$\begin{aligned} &\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_i))\|_1^{\Theta}\right) + \left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_j))\|_1^{\Theta}\right) \\ = &\left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_1^{\Theta}\right) + \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_1^{\Theta}\right). \end{aligned} \tag{3.11}$$

(ii.c) Finally, following the conditions of $\mathcal{ERA}$, let $\iota(h_{\mathbf{r}}(p_i)) = 1$ and $\iota(\tilde{h}_{\mathbf{r}}(p_j)) = 0$ such that $\Upsilon(\mathbf{H}_{-\mathbf{r}}) = \Upsilon(\widetilde{\mathbf{H}}_{-\mathbf{r}}) = [\mathbf{0}]$. We have $\lim_{\alpha\to\infty}\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_i))\|_\alpha^{\Theta}\right) = \lim_{\alpha\to\infty}\left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_\alpha^{\Theta}\right) = 0$. Also, $\lim_{\alpha\to\infty}\left(1 - \|\Upsilon(\widetilde{\mathbf{H}}(p_j))\|_\alpha^{\Theta}\right) = 1$ and $\lim_{\alpha\to\infty}\left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^{\Theta}\right) = 1$, then the same contradiction occurs. ■

The previous results shows that $\mathcal{CIR}$ is relevant with risk aversion only, provided that $\alpha > 1$ is not in the neighborhood of infinity.[5] Also, result (i) indicates that the analysis in terms of risk aversion is general since it can be done with a bounded variable $\iota(h_k(p)) \in [0, 1]$ instead of a Boolean one Eq.(2.1). This means that the use of the threshold τ_k can be relaxed.

3.4 Snowballing risk effect

Let us now investigate the snowballing risk effect by denoting $\phi_\alpha(\mathbf{H}_p(\mathcal{S}))$ the achievement of the agent at rank p being non healthy in some health dimensions represented by the set $\mathcal{S} \subset \mathcal{K}_{-\mathbf{r}}$. Consider that individual p_1 is endowed with $|\mathcal{S}|$ non healthy dimensions whereas individual p_2 is endowed with $|\mathcal{R}|$ non healthy dimensions such that $|\mathcal{R}| = |\mathcal{S}| + 1 < |\mathcal{K}|$. Adding to each individual the same risk factor $\mathbf{r}$ could imply that the increase in the non-healthy situation of individual p_2 is deeper than p_1. In such a case,

$$\begin{aligned} &\|\Upsilon(\mathbf{H}_{p_1}(\mathcal{S} \cup \{\mathbf{r}\}))\|_\alpha^{\Theta} - \|\Upsilon(\mathbf{H}_{p_1}(\mathcal{S}))\|_\alpha^{\Theta} \\ < \; &\|\Upsilon(\mathbf{H}_{p_2}(\mathcal{R} \cup \{\mathbf{r}\}))\|_\alpha^{\Theta} - \|\Upsilon(\mathbf{H}_{p_2}(\mathcal{R}))\|_\alpha^{\Theta}. \end{aligned} \tag{3.12}$$

In the sequel, we will say that $\widetilde{\mathbf{H}}$ is issued from $\mathbf{H}$ by a snowballing risk effect involving two persons p_i and p_j, if their health situations are described as above, that is, an additional risk factor is added to both p_i and p_j with $\widetilde{\mathbf{H}}$ being associated with more non-healthy dimensions than $\mathbf{H}$.

Property 3.3 – Snowballing Risk Effect – $\mathcal{SRE}$:
For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, if $(\widetilde{\mathbf{H}}, \mathbf{H}) \in \mathcal{SRE}$ such that $\widetilde{\mathbf{H}}$ is issued from $\mathbf{H}$ by a snowballing risk effect involving two persons p_i and p_j, then:

$$I_{\nu,\alpha}(\widetilde{\mathbf{H}}) > I_{\nu,\alpha}(\mathbf{H}).$$

[5] It is noteworthy that relaxing the condition on the mean would imply $\mu_{\phi(\mathbf{H})} < \mu_{\phi(\tilde{\mathbf{H}})}$.

This property is well-used in the field of cooperative game theory, *i.e.* convex games. It postulates that the risk factor provides an acceleration of the non-healthy state of an individual p_i who is initially less healthy than another individual p_j.

Lemma 3.2 *For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, let $(\widetilde{\mathbf{H}}, \mathbf{H}) \in \mathcal{SRE}$ for two individuals p_i and p_j such that $p_i = p_j$. Then the two following statements hold.*
(i) *Assume that the aggregator $\|\cdot\|_\alpha^\Theta$ is approximated by a twice differentiable function $f : [0,1]^K \longrightarrow [0,1]$. Then, $I_{\nu,\alpha}$ respects $\mathcal{SRE} \iff \|\cdot\|_\alpha^\Theta$ is strictly L-superadditive and $\alpha > 1 \iff I_{\nu,\alpha}$ respects $\mathcal{CIR}$.*
(ii) *Whenever $\alpha \in \{0, 1, \infty\}$, it results that:*

(a) $\mathcal{SRE} \cap \mathcal{RN} = \emptyset$.

(b) $\mathcal{SRE} \cap \mathcal{RA} = \emptyset$ *whenever* $\alpha = 1$.

(c) $\mathcal{SRE} \cap \mathcal{ERA} = \emptyset$.

Proof:
(i) $\mathcal{SRE}$ implies that:

$$\|\Upsilon(\mathbf{H}(p_i))\|_\alpha^\Theta + \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^\Theta < \|\Upsilon(\widetilde{\mathbf{H}}(p_i))\|_\alpha^\Theta + \|\Upsilon(\widetilde{\mathbf{H}}(p_j))\|_\alpha^\Theta. \tag{3.13}$$

Let $f(x) = \|x\|_\alpha^\Theta$ such that $f : [0,1]^K \longrightarrow [0,1]$ is twice differentiable. The last expression becomes, for $\delta \in [0,1]$ and setting $|\mathcal{R}| = |\mathcal{S}| + 1$:

$$\begin{aligned} & f(x_1, \ldots, x_{|\mathcal{S}|}, 0, \ldots, \delta) - f(x_1, \ldots, x_{|\mathcal{S}|}, 0, \ldots, 0) \\ < \; & f(x_1, \ldots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \ldots, \delta) - f(x_1, \ldots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \ldots, 0). \end{aligned} \tag{3.14}$$

Now let $x_{\mathcal{S}} := (x_1, \ldots, x_{|\mathcal{S}|}, 0, \ldots, 0)$, $x_{\mathcal{R}} := (x_1, \ldots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \ldots, 0)$ and let $\delta = x_K \to 0$, thus dividing both sides of the last expression provides:

$$\frac{\partial f(x_{\mathcal{S}})}{\partial x_K} < \frac{\partial f(x_{\mathcal{R}})}{\partial x_{\mathbf{r}}}. \tag{3.15}$$

Since $x_{\mathcal{R}} = x_{\mathcal{S}} + (0, \ldots, 0, x_{|\mathcal{R}|}, 0, \ldots, 0)$, then letting $x_{|\mathcal{R}|} \to 0$ and dividing both sides of the previous expression by $x_{|\mathcal{R}|}$ entails:

$$0 < \frac{\partial^2 f(x_{\mathcal{R}})}{\partial x_{\mathbf{r}} \partial x_{|\mathcal{R}|}}. \tag{3.16}$$

As in Lemma 3.1, the last condition is fulfilled whenever $\alpha > 1$. In this respect, we obtain the same implications compared with $\mathcal{CIR}$, *i.e.*, strict L-superaddivity, so that $\mathcal{SRE}$ and $\mathcal{CIR}$ are equivalent whenever $\alpha > 1$.

(ii) Points (a), (b) and (c) provide contradictions in the same manner as in Lemma 3.1 (ii.a), (ii.b) and (ii.c) since in those cases:

$$\begin{aligned} & f(x_1, \ldots, x_{|\mathcal{S}|}, 0, \ldots, \delta) - f(x_1, \ldots, x_{|\mathcal{S}|}, 0, \ldots, 0) \\ = & f(x_1, \ldots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \ldots, \delta) - f(x_1, \ldots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \ldots, 0). \end{aligned} \tag{3.17}$$

■

The last result comes as a surprise since the restriction $\alpha > 1$ (with α that does not tend to ∞) allows for two apparently different properties to be matched, both $\mathcal{CIR}$ and $\mathcal{SRE}$. However, nothing is said about extreme inequality aversion when $\alpha \to \infty$. In the sequel, we formalize this last property.

3.5 Critical risk level

Consider a distribution $\mathbf{H} \in \mathcal{H}$, such that $1 = \iota(h_{\mathbf{r}}(p_i)) \neq \iota(h_{\mathbf{r}}(p_j)) = 0$, from which we derive two distributions $\mathbf{H}_1$ and $\mathbf{H}_2$ by focusing on two individuals p_i and p_j only. On the one hand, the former is derived from $\mathbf{H}$ such that the worst health dimensions faced by both p_i and p_j are gathered, in particular,

$$\|\Upsilon(\mathbf{H}_1(p_{ij}))\|_\alpha^\Theta := \|\Upsilon(\mathbf{H}(p_i)) \vee \Upsilon(\mathbf{H}(p_j))\|_\alpha^\Theta . \tag{3.18}$$

More precisely, $\mathbf{H}_1$ is obtained from $\mathbf{H}$ by replacing individuals p_i and p_j by p_{ij} defined above, with no change for the other individuals. On the other hand, the latter is given by the worst situation between individuals p_i and p_j, that is, the maximum (bad health) situation between p_i and p_j:

$$\|\Upsilon(\mathbf{H}_2(p_{ij}))\|_\alpha^\Theta := \bigvee_{r=i,j} \|\Upsilon(\mathbf{H}(p_r))\|_\alpha^\Theta . \tag{3.19}$$

Again, $\mathbf{H}_2$ is obtained from $\mathbf{H}$ by replacing individuals p_i and p_j by p_{ij}, *ceteris paribus*. In the sequel, we will say by Eq.(3.18) and Eq.(3.19) that the distributions $\mathbf{H}_1$ and $\mathbf{H}_2$ are derived from $\mathbf{H}$ by a rearrangement of a critical risk level. The property is the following.

Property 3.4 – Critical Risk Level – $\mathcal{CRL}$:
For all inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, if $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ such that $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ are issued from $\mathbf{H}$ by a rearrangement of a critical risk level between two individuals p_i and p_j, then:

$$I_{\nu,\alpha}(\mathbf{H}_1) = I_{\nu,\alpha}(\mathbf{H}_2).$$

This property is actually matched whenever the risk dimension $\mathbf{r}$ is associated with a critical risk level, such as irreversible diseases. Hence, in a given distribution $\mathbf{H}$, replacing two individuals p_i and p_j by one virtual individual p_{ij} who takes on all their non healthy dimensions (including the irreversible risk factor), or by one virtual individual who takes on their irreversible risk dimension only, yields, *ceteris paribus*, exactly the same socio-economic health inequality index. This is because the dimension associated with critical risk dominates the other ones. Consequently, for an individual ranked p affected by a critical risk factor, his bad health situation is maximum whatever the values of the other dimensions. This is in line with the property of extreme risk aversion $\mathcal{ERA}$. However $\mathcal{CRL}$ is weaker than $\mathcal{ERA}$, since $\mathcal{ERA}$ applies for matrices of zeros $\Upsilon(\mathbf{H}_{-\mathbf{r}})$ and $\Upsilon(\tilde{\mathbf{H}}_{-\mathbf{r}})$, whereas this is not systematically the case for the critical risk level property. The equivalence is given below.

Lemma 3.3 *For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, let $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ with $p_i = p_j$. Then, the two following statements are equivalent.*
(i) *$I_{\nu,\alpha}$ satisfies extreme risk aversion $\mathcal{ERA}$.*
(ii.a) *$I_{\nu,\alpha}$ satisfies critical risk level $\mathcal{CRL}$ and,*
(ii.b) *for all $\mathbf{H} \in \mathbb{R}_+^K$ and $t \in [0,1]$, $\|t\Upsilon(\mathbf{H})\|_\alpha^\Theta = t\,\|\Upsilon(\mathbf{H})\|_\alpha^\Theta$ such that $\theta_K = 1$.*

Proof:
[(ii) $\Longrightarrow$ (i)]: For simplicity, set $f(x) := \|x\|_\alpha^\Theta$ such that $x := \Upsilon(\mathbf{H})$. From (ii.a), equations (3.18) and (3.19) are equivalent *i.e.*:

$$\|\Upsilon(\mathbf{H}_1(p_{ij}))\|_\alpha^\Theta = \|\Upsilon(\mathbf{H}(p_i)) \vee \Upsilon(\mathbf{H}(p_j))\|_\alpha^\Theta = \bigvee_{r=i,j} \|\Upsilon(\mathbf{H}(p_r))\|_\alpha^\Theta = \|\Upsilon(\mathbf{H}_2(p_{ij}))\|_\alpha^\Theta\,,$$

that is, setting $x_i := \Upsilon(\mathbf{H}(p_j))$ and $x_j := \Upsilon(\mathbf{H}(p_j))$,

$$f(x_i \vee x_j) = \max\{f(x_i), f(x_j)\}. \tag{3.20}$$

From (ii.b), since $\|\mathbf{0}\|_\alpha^\Theta = 0$, it results that,

$$f(tx) = \max\{tf(x), 0\}. \tag{3.21}$$

From Briec and Horvath (2004, Proposition 3.0.3.), equations (3.20) and (3.21) hold if, and only if,

$$f(x) = \max_{k=1,\dots,K} \theta_k x_k \ ; \ \forall x, \Theta \in \mathbb{R}_+^K. \tag{3.22}$$

Then, setting $\theta_K = 1$ and choosing a distribution $\widetilde{\mathbf{H}}$ issued from $\mathbf{H}$ such that $\Upsilon(\mathbf{H}_{-\mathbf{r}}) = [\mathbf{0}]$ and $\iota(h_{\mathbf{r}}(p)) = 1, \iota(\tilde{h}_{\mathbf{r}}(p)) = 0$, then Eq.(3.22) implies that: $\phi_\infty(\mathbf{H}(p)) = 0 < 1 = \phi_\infty(\widetilde{\mathbf{H}}(p))$.

[(i) $\Longrightarrow$ (ii)]: From $\mathcal{ERA}$, $\|x\|_{\alpha}^{\Theta} = \max_{k=1,\ldots,K} x_k$. In this case, choosing $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ with $p_i = p_j$, implies that $\|\Upsilon(\mathbf{H}_1(p_{ij}))\|_{\infty}^{\Theta} = \|\Upsilon(\mathbf{H}_2(p_{ij}))\|_{\infty}^{\Theta}$, then $I_{\nu,\infty}(\mathbf{H}_1) = I_{\nu,\infty}(\mathbf{H}_2)$, which concludes the proof. ■

Finally, the socioeconomic health inequality indices are endowed with various attitudes to risk. The different properties are summarized in the following proposition.

Proposition 3.1 *For all socioeconomic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$:*
(i) *$I_{\nu,\alpha}$ satisfies $\mathcal{RN}$, if $\alpha \to 0$.*
(ii) *$I_{\nu,\alpha}$ satisfies $\mathcal{RA}$, $\mathcal{CIR}$, $\mathcal{SRE}$, if $\alpha > 1$.*
(iii) *$I_{\nu,\alpha}$ satisfies $\mathcal{ERA}$ and $\mathcal{CRL}$, if $\alpha \to \infty$.*

Proof:
See Lemma 3.1 to Lemma 3.3. ■

Though risk sensitivity is now captured by the α parameter, nothing has been said about distributional sensitivity, which is introduced and discussed in the next section.

4 Boolean Risk Factor and Stochastic Dominance Criteria

In this section, the interplay between the parameters ν and α is presented in order to figure out the class of measures $\{I\}_{\nu,\alpha}$. Particularly, it shows the role of ν in determining the willingness of the social planner to perform health redistribution. Lastly, the result in terms of stochastic dominance is set out.

4.1 Distributional sensitivity

Distributional sensitivity may also be captured by the function $v(p)$, which provides the behavior of the social planner with respect to health transfers between individuals at *different rank levels*, instead of looking for permutations between the situations of individuals. However, in the same manner, this kind of distributive sensitive principle enables one to choose whether the decision maker has to implement transfers towards non healthy people, towards very non healthy people, or to the most non healthy ones. Those transfers are actually well-documented in the literature. Aaberge (2009) shows, for Yaari's (1987) dual social welfare function, the conditions needed for respecting the s-th degree positional (income) transfer principle. When

$\nu = 1$, an exogenous manna for one person improves his welfare and the overall welfare therefore also increases. The principle of order 2 postulates that the overall welfare increases if a rich-to-poor transfer occurs between an income donor at rank p_1 and an income recipient at p_2 such that $p_1 > p_2$. Those transfers are generalized in such a way that more weights are put on the lower income recipient insofar as s increases. Formally, in the case of health inequality indices, the property of ν-th degree of the positional transfer principle is captured when the weight function $v(p)$ has derivatives that alternate in signs, see Makdissi and Yazbeck (2014). The social planner performs some health transfers between the individuals. The set of socio-economic health inequality indices that respect the ν-th degree positional transfer principle is:

$$\Omega^{\nu} := \left\{ I_{\nu,\alpha}(\mathbf{H}) \in \{I\}_{\nu,\alpha} \;\middle|\; \begin{array}{l} v^{(\ell)} \text{ is continuous and } \nu\text{-time differentiable over } [0,1] \\ (-1)^{\ell} v^{(\ell)}(p) \geq 0 \ \forall p \in [0,1] \ ; \forall \ell = 1, \ldots, \nu - 1 \\ v^{(\ell)}(1) = 0, \quad ; \forall \ell = 1, \ldots, \nu. \end{array} \right\}$$

In order to deal with the generalized Gini indices of inequality the weight may be restricted to $v(p) = \nu(1-p)^{\nu-1}$, such that $\nu = s$ in order to respect the ν-th positional transfer principle for $\nu \geq 2$. In that case, the social planner is more and more inequality averse to the extent that s increases. On the contrary, he is inequality loving whenever $\nu \in (1, 2)$, or neutral to inequality if $\nu = 1$. The sensitivity to risk and to inequality may be summarized in the following Figure 0.

Figure 0 – The family $\{I\}_{\nu,\alpha}$

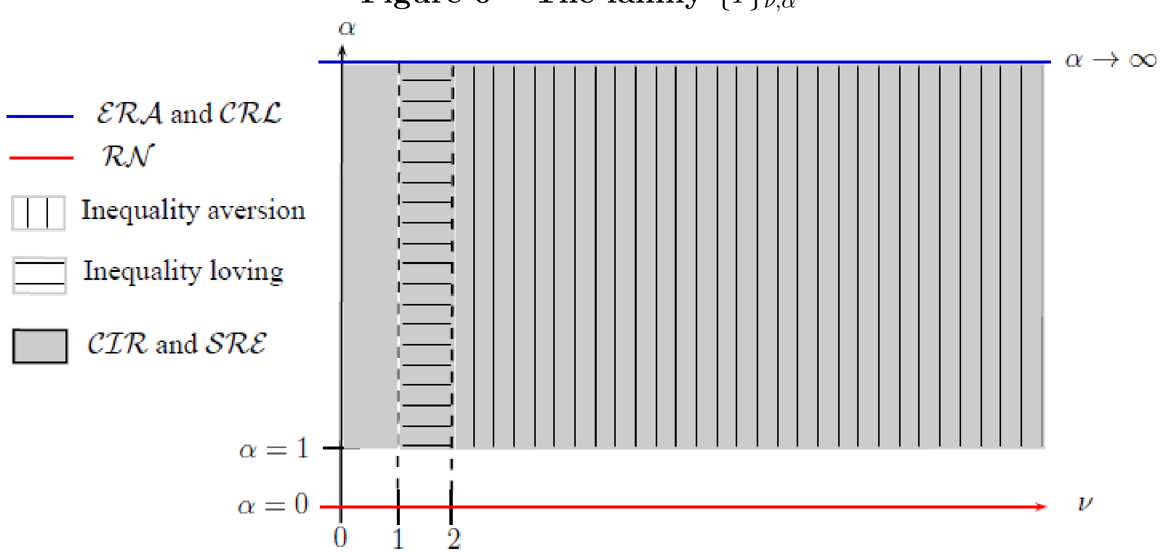

Figure 0 exhibits a snapshot of two distributional types of sensitivities: inequality aversion (variations of ν) and risk aversion (variations of α). In this way, the parametrization of the socio-economic health inequality indices may be performed with respect to the different properties inherent to ν and α, that is, according to risk and inequality sensitivities. In the same manner, in Table 1 below, the properties and all possible parametrizations are summarized for the two-parameter family of socio-economic health inequality indices $\{I\}_{\nu,\alpha}$.

Table 1. Properties of $I_{\nu,\alpha}$

$\nu \downarrow \quad \alpha \rightarrow$	$\alpha = 0 : \mathcal{RN}$	$\alpha > 1 : \mathcal{RA}$	$\alpha \rightarrow \infty : \mathcal{ERA}$
Inequality loving $\nu \in (1,2)$	$\emptyset$	$\mathcal{CIR} \cup \mathcal{SRE}$	$\mathcal{CRL}$
Inequality neutrality $\nu = 1$	$\emptyset$	$\mathcal{CIR} \cup \mathcal{SRE}$	$\mathcal{CRL}$
Inequality aversion $\nu \geq 2$	$\emptyset$	$\mathcal{CIR} \cup \mathcal{SRE}$	$\mathcal{CRL}$

$\emptyset$: no particular property of risk (outlined in the paper)

4.2 Stochastic dominance result

In this section, we show that, without imposing any functional form on $v(p)$, but taking recourse to a multidimensional socio-economic health inequality index $I_{\nu,\alpha}(\mathbf{H}) \in \Omega^{\nu}$, we can derive a non-ambiguous ranking between health distributions. Contrary to the previous papers in the literature, see *e.g.* Makdissi and Yazbeck (2014) or Mussard, Pi Alperin and Thireau (2016), we propose concentration curves, more precisely, achievement curves, which involve the risk attitude of the social planner. For that purpose, let us introduce the achievement curve of order (ν, α), with $\nu \in \{1, 2, 3, \ldots\}$ and $\alpha > 0$.

Definition 4.1 – **ν-Achievement curves** – *The achievement curve of order $(1, \alpha)$ is defined as, for all $\alpha > 0$:*

$$A^{1,\alpha}_{\mathbf{H}}(p) := \frac{1 - \|\Upsilon(\mathbf{H}(p))\|^{\Theta}_{\alpha}}{\mu_{\phi(\mathbf{H})}}. \tag{4.1}$$

The (ν, α)-order achievement curve for any given $\nu \in \{2, 3, \ldots\}$ and $\alpha > 0$ is:

$$A^{\nu,\alpha}_{\mathbf{H}}(p) := \int_0^p A^{\nu-1,\alpha}_{\mathbf{H}}(u)\, du. \tag{4.2}$$

In proportion of the mean, the achievement curve of order $(1, \alpha)$ yields the health achievement of one individual at rank p of the population. The second-order achievement curve $(2, \alpha)$ provides the proportion $p\%$ of the population whose health achievement is no higher than $A^{2,\alpha}_{\mathbf{H}}(p)$. Accordingly, a dominance criterion between achievement curves yields a non-ambiguous ranking between health distributions (say $\mathbf{H}$ and $\mathbf{G}$). As a result, when the curves do not cross, there is more (or less) inequality for all percentiles $p \in [0,1]$. Also, this result is in line with the bidimensional parametrization highlighted in the previous section.

Theorem 4.1 *For all socio-economic health inequality indices $I_{\nu,\alpha}(\mathbf{H}) \in \{I\}_{\nu,\alpha}$ such that $I_{\nu,\alpha}(\mathbf{H}) \in \Omega^{\nu}$ with $\nu \in \{1,2,3,\ldots\}$, $\alpha > 0$, and for two health distributions $\mathbf{H}$ and $\mathbf{G}$, the two following statements are equivalent:*

(i) $I_{\nu,\alpha}(\mathbf{H}) \geq I_{\nu,\alpha}(\mathbf{G})$.

(ii) $A^{\nu;\alpha}_{\mathbf{H}}(p) \leq A^{\nu;\alpha}_{\mathbf{G}}(p)\,,\ \forall p \in [0,1]$.

Proof:

See the appendix. ■

Theorem 4.1 provides a non-ambiguous ranking for multidimensional health distributions with respect to various attitude to risk (neutrality $\alpha \to 0$, aversion $\alpha > 1$, and extreme aversion $\alpha \to \infty$). However, the non-ambiguous ranking comes at a cost since the ν parametrization becomes discrete only. As a consequence, inequality loving ($\nu \in (1,2)$) is not available.[6] The different dominance criteria and their link with the risk properties defined in Section 3 are itemized in Table 2 below.

Table 2. Dominance criteria and properties of $I_{\nu,\alpha}$

$\nu \downarrow \quad \alpha \to$	$\alpha = 0 : \mathcal{RN}$	$\alpha > 1 : \mathcal{RA}$	$\alpha \to \infty : \mathcal{ERA}$
Inequality loving $\nu \in (1,2)$	Impossible	Impossible	Impossible
Inequality neutrality $\nu = 1$	$\emptyset$	$\mathcal{CIR} \cup \mathcal{SRE}$	$\mathcal{CRL}$
Inequality aversion $\nu \geq 2$	$\emptyset$	$\mathcal{CIR} \cup \mathcal{SRE}$	$\mathcal{CRL}$

$\emptyset$: no particular property of risk (outlined in the paper)

5 Empirical application

This paper uses data from wave 5 Release 1.0.0 of the Survey of Health, Ageing and Retirement in Europe (SHARE; Börsch-Supan, Brandt, Hunkler, Kneip, Korbmacher, Malter, Schaan, Stuck and Zuber, 2013; Börsch-Supan, 2015, and Malter and Börsch-Supan, 2015). SHARE is a multidisciplinary and cross-national panel database collecting micro data on health, socio-economic status and social and family networks. The objective of the survey

[6] Inequality loving may be captured when the derivatives of $v(p)$ alternate in opposite signs $(-1)^{\ell}\, v^{(\ell)}(p) \leq 0$.

is to better understand the ageing process and, in particular, to examine the different ways in which people aged 50 and older live in Europe. The first wave of data was collected in 2004. Wave 5 was collected in 2013 in fourteen European countries (Austria, Belgium, Czech Republic, Denmark, Estonia, France, Germany, Italy, the Netherlands, Luxembourg, Slovenia, Spain, Sweden, and Switzerland) and Israel.

The analysis is focused on the individuals living in Luxembourg. After excluding all individuals with missing values on any of the variables used in our empirical analysis, our estimation sample includes 1,610 individuals aged 50 and older and their partners. Within this sample, 47% are males and 57% are over 65 years old.

5.1 Health dimensions

Synthetic indicators of health have been constructed following the methodology proposed by Pi Alperin (2016). These indicators aggregate nine single or composite dimensions reflecting different aspects of the mental and physical health status of the individuals (see Table 3). In particular, this methodology allows to relax the threshold τ_k (from Eq. (2.1)), in order to allow different degrees of 'deprivation' for each dimension of health. Consequently, for each dimension there are healthy individuals, completely non healthy individuals and individuals characterized by different intensities of health failure.[7] More precisely, the synthetic scores are calculated as the weighted mean of the K dimensions of health. In order to determine the weight vector Θ, we use the 'equal weighting' scheme, for which each dimension of health has the same importance in the final score, that is, $1/K$.[8]

Table 3. Health dimensions

Global health	Dimensions of health	What is covered by the dimension of health?
Mental Health	Depression	Twelve different aspects of a depression's symptoms
	Memory	The ability of people to think about things
Physical health	Long term illness	Having any long-term health problem, illness or infirmity
	Other illnesses	A list of fourteen health conditions
	Limitation activities 1	Difficulties with various activities because of health problems
	Limitation activities 2	Difficulties with various basic daily activities
	Weight problems	Overweight, obesity and underweight problems
	Eyesight	Eyesight distance and reading
	Hearing	Quality of hearing (with or without hearing aid)

[7] See Appendix A.2 for a complete description of the construction of each dimension.

[8] All the indicators are computed using the MDEPRIV program (see Pi Alperin and Van Kerm, 2009).

5.2 Exogenous risk

A growing number of articles try to distinguish between legitimate and illegitimate causes of health inequalities (see for instance Rosa-Dias 2009, Trannoy *et al.* 2010, Rosa-Dias 2010, Garcia Gomez *et al.* 2015, Jusot, Tubeuf, and Trannoy 2013). The main idea behind this distinction is the fact that the health outcomes can be the consequence of circumstances that are beyond an individual's control, and autonomous choices which are within his/her control. In his article, Roemer (1995) recommends that society compensates only the cases where bad consequences are due to circumstances or brute luck.[9] In other words, these circumstances can be identified as exogenous risks that can increase inequalities in health, these inequalities that should be compensated.

In our empirical application four explanatory variables were assumed to be part of the individual's circumstances: the nationality of parents, the educational level of parents, the economic situation of the family during childhood and the longevity of parents. In particular, these variables have been selected as they are frequently used to measure childhood conditions circumstances that can have an influence in adults' health outcomes. For example, Deutsch, Pi Alperin and Silber (2016) show that in Luxembourg, the probability of having good health is higher among natives, the higher the educational level of the father, and for those who did not have financial difficulties when they were young.Jusot *et al.* (2013) use the longevity of parents as a proxy for parents' health.

The nationality of the mother and the father can be considered as a circumstance variable which can eventually impact on an individual's health status. This is especially important in Luxembourg since the countryrecords the largest share of immigrants in the European Union (European Commission, 2011). In particular, the variable nationality of the parents' set to 1 assigned to an individual for whom both parents are immigrants and to zero otherwise. Almost 65% of individuals have at least one parent with the Luxembourgish nationality.

As regard the educational level of each parent, a distinction was made between two categories: those who have either no education, a primary or a lower secondary education, and those who studied beyond high school. Subsequently, the education level of parents' risk factor is computed to be equal to 1 if both parents belong to the first group of no education, primary or lower secondary education. In contrast, if one of the parents studied beyond high school, then the risk value is equal to 0. Only 34,41% of the individuals

[9] Dworkin (2000) defined brute luck as a situation which happens when, for example, you are hit by a car that jumped a red light and you walked in the pedestrian crossing.

have at least one of their parents who have further education.

The third variable included as individuals' circumstances was the economic situation of the family during childhood. Individuals were thus asked whether their family used to have financial difficulties when they were growing up, from birth to age 15 inclusive. This variable was assumed to take two values: for those who answered pretty well, or about average, the variable was equal to 0, while for those from poor families or families whose financial situation varied over time, the variable is equal to 1. This variable does not represent a risk for more than 75% of the population.

The last risk factor concerns the parent's longevity. Individuals in the survey report whether the parents are still alive at the time of the survey and their age at death if applicable. With this information the longevity risk factor was set up to be equal to one if at least one of the parents had short longevity (i.e. those who died younger than the life expectancy of their generation at birth) and equal to 0 for those individuals with both parents enjoying longevity. Only 16% of the population have one, or both of their parents, in the category short longevity.

5.3 Risk neutrality

If the decision maker is risk neutral, then for any given exogenous risk factor associated with the health dimensions, the socio-economic health inequality index remains the same. In other words, the (absolute) achievement curves remain (almost) invariant with respect to the considered risk factors. In that case, as can be seen in the following figures, the results are in conformity with the theoretical prediction.

More precisely, at the order 2 ($\nu = 2$), it is possible to see that the achievement curve is close to the 45° line, meaning that the socio-economic health inequality in the society is the lowest possible, *i.e.* the aggregated health information is equally distributed among the individuals of the society. Indeed, a risk neutral decision-maker considers an individual being non-healthy, if he is non-healthy in all dimensions – the union approach. In addition, at the order 2, the decision-maker respects the Pigou-Dalton principle of transfers (a Daltonean decision-maker), then he could operate health transfers even if he is risk neutral. This would be the case for the individuals being non-healthy in all dimensions including the risky one.

At the order 3 ($\nu = 3$), the same remarks hold true and the risk factors cannot be distinguished since the decision maker is risk neutral. However, he is more inclined to perform redistribution towards non-healthy people, since Kolm's transfer principle is respected. Also, at the order 3, all the curves shift down, showing a higher aversion towards inequality.

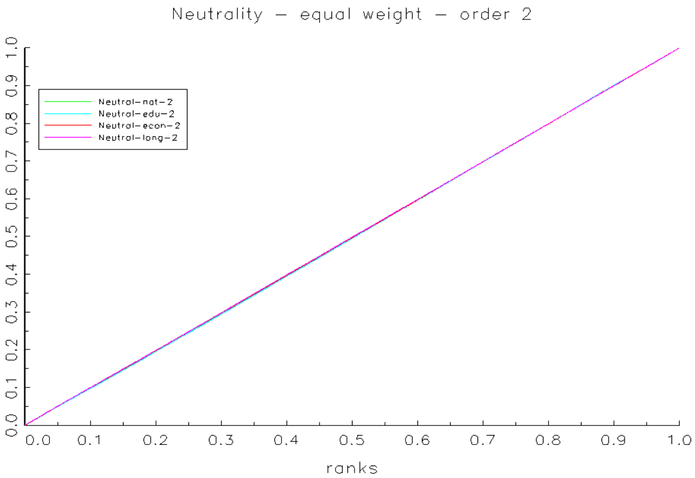

Figure 1a: Order 2, $\alpha = 0$

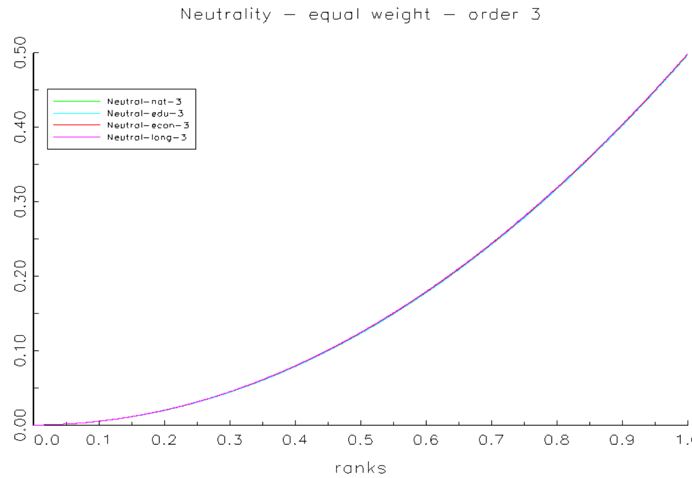

Figure 1b: Order 3, $\alpha = 0$

5.4 Risk aversion

There are different degrees of risk aversion ranging from a low risk aversion to an important one. This distinction can be made by imposing different values to the α parameter in Eq. (2.1), α going from 1 to infinity.

Figures 2a-2d show the results of imposing different values to α. On the one side, it is possible to see that the higher the value of α, the further the curves to the 45° line are, showing that the more averse to risk is the decision-maker, more important is the impact of the risk factors on socio-economic health inequalities. On the other side, we can see that the decision-maker cannot proceed to a non-ambigous ranking of the health matrices associated with different types of exogenous risk factors. This is because the curves cross, so that a Daltonean decision-maker could not say with certainty whether the bad education of the parents or their nationality is the main cause of the non-healthy state of the individuals, implying a high socio-economic health inequality in the society.

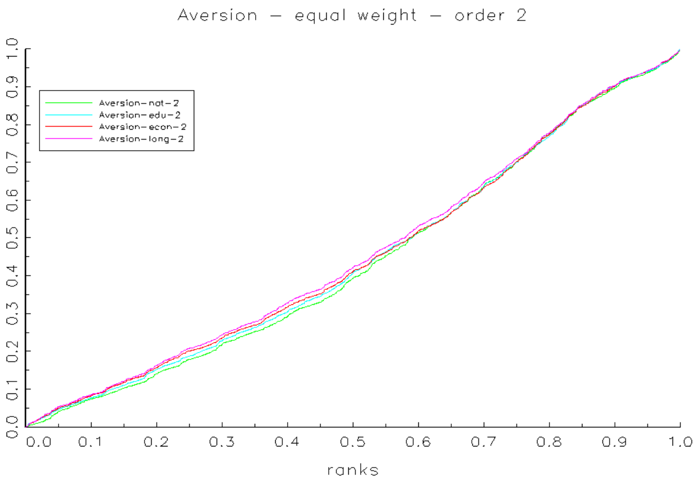

Figure 2a: Order 2, $\alpha = 20$

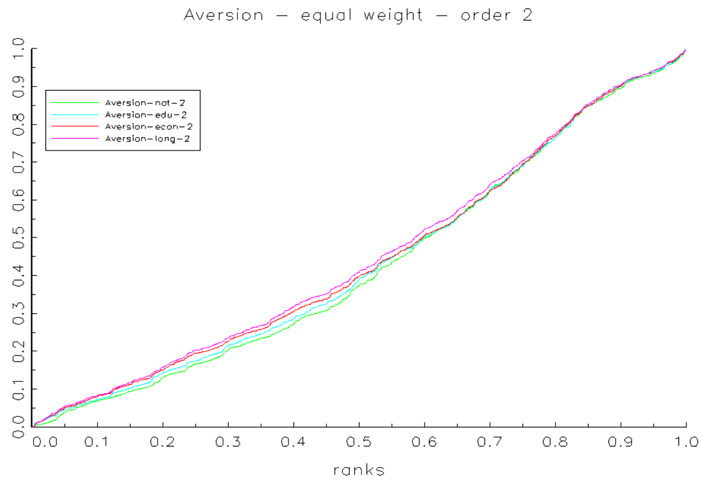

Figure 2b: Order 2, $\alpha = 30$

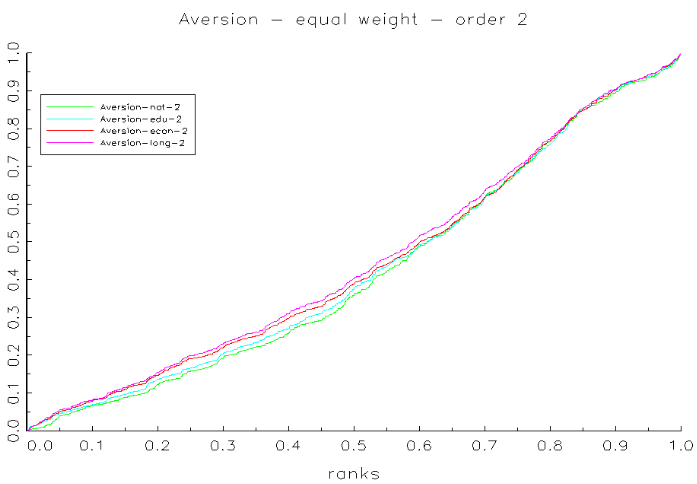

Figure 2c: Order 2, $\alpha = 40$

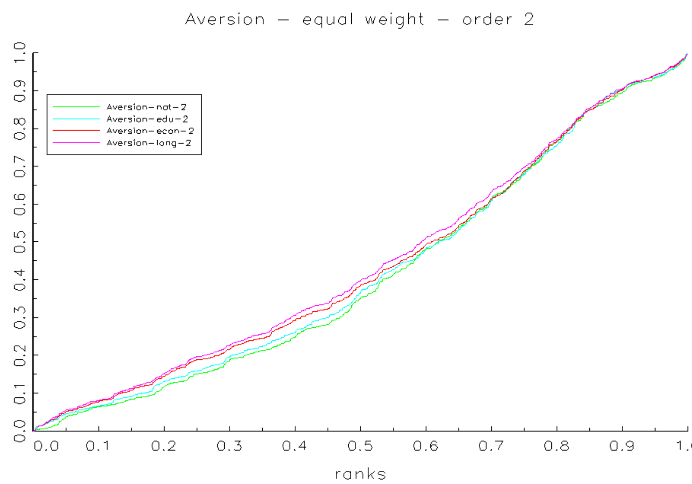

Figure 2d: Order 2, $\alpha = 50$

At the order 3 ($\nu = 3$), the curves do not cross when the degree of risk aversion increases. A Kolm decision-maker, being risk averse, non-ambiguously ranks the distribution with the nationality risk factor as the most unequal one followed by education, economic situation and longevity risk factors. In other words, among the different circumstances the agents do not control for, having both parents as immigrants constitutes the risk factor that most aggravates the overall level of inequality.

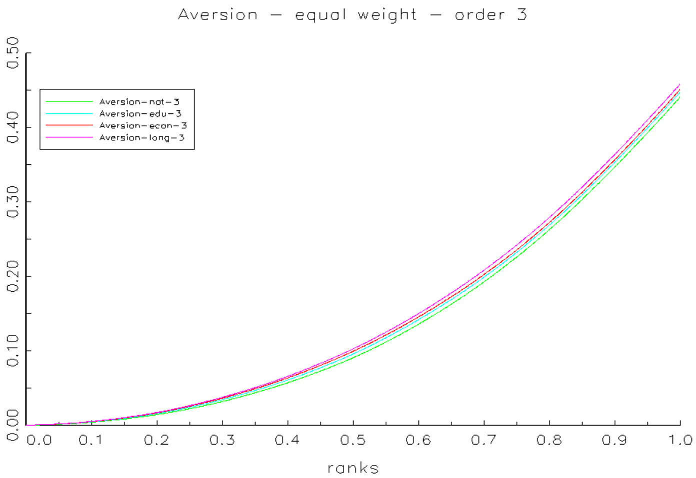

Figure 3a: Order 3, $\alpha = 20$

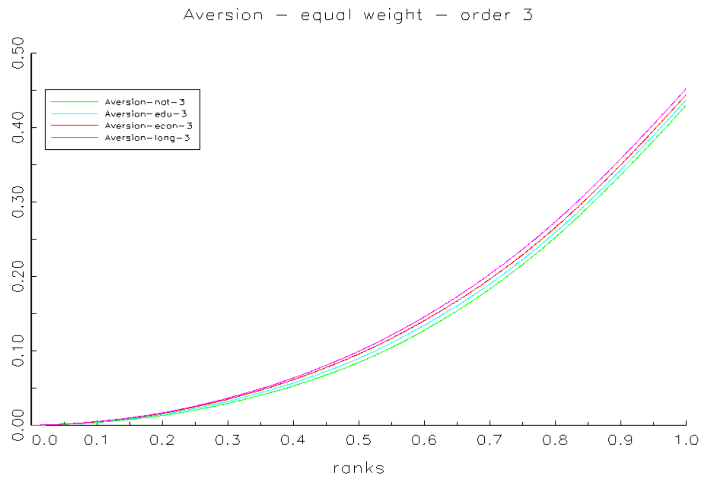

Figure 3b: Order 3, $\alpha = 30$

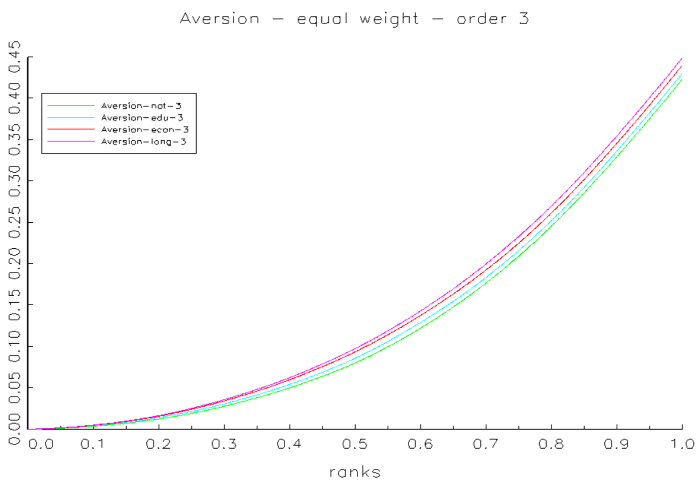

Figure 3c: Order 3, $\alpha = 40$

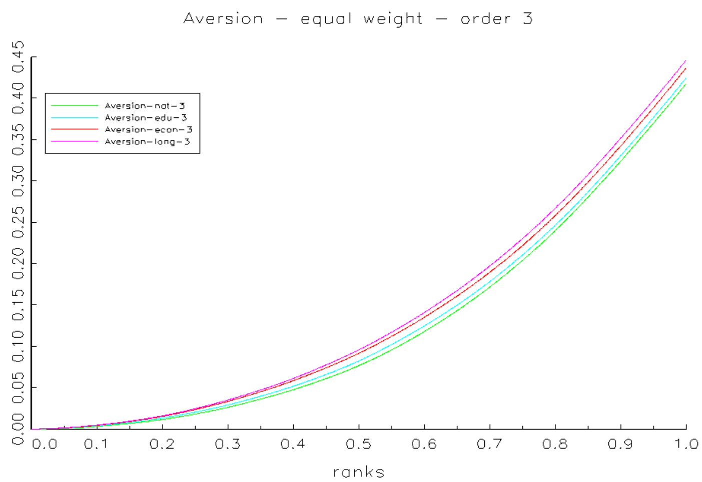

Figure 3d: Order 3, $\alpha = 50$

5.5 Extreme risk aversion

Three different orders have been computed in the case of extreme risk aversion in order to have a clear distinction between the four risk factors. At the order 2 ($\nu = 2$), the achievement curves cross and it becomes difficult to identify those risk factors that increase socio-economic health inequality. At the order 3 ($\nu = 3$), the extreme aversion case identifies the economic situation as the riskiest factor followed by the longevity one. Nationality and education cannot be distinguished since the achievement curves cross at the order 3. At the order 4 ($\nu = 4$), for a decision-maker that respects composite transfers, the health information matrix associated with the risk nationality provides a $(4, \alpha)$-order achievement curve that slightly dominates that of the educational risk factor. In this respect, nationality is the most risky factor that contributes to the overall socio-economic health inequality in a society.

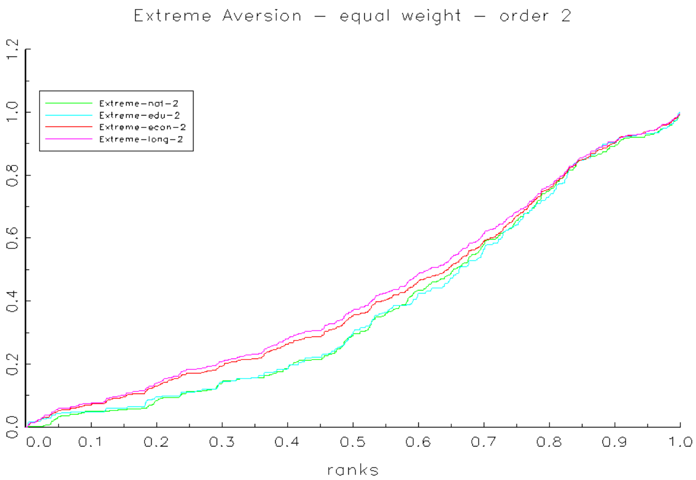

Figure 4a: Order 2, $\alpha = \infty$

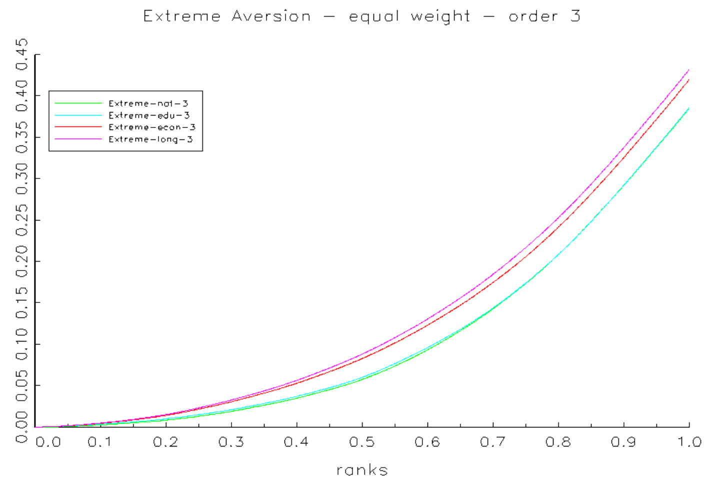

Figure 4b: Order 3, $\alpha = \infty$

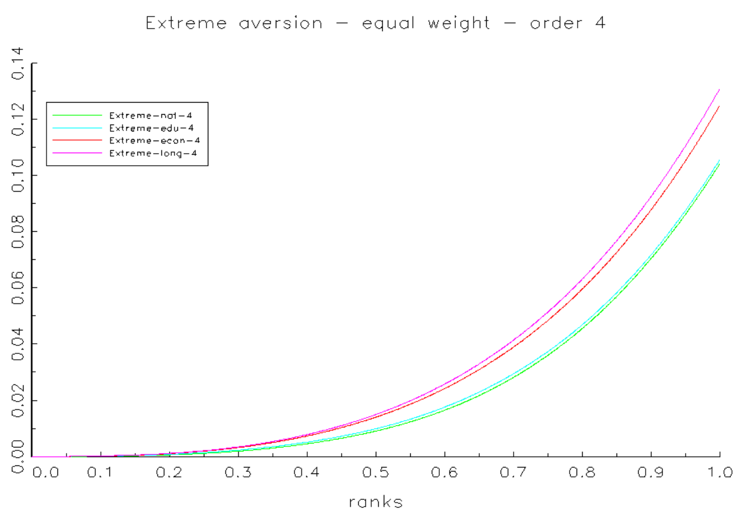

Figure 4c: Order 4, $\alpha = \infty$

5.6 Risk mixture

As mentioned in Section 3, the socio-economic health inequality indices may be computed with respect to a bounded risk factor $h_{\mathbf{r}}(p) \in [0,1]$, which can be defined as the mean of all risk factors (Risk_mix). This variable is interesting because it enables the risky factors to be compared with a mean risk. In our paper, it allows the health information matrix associated with the nationality of the parents to be compared with the health information in which an average risk factor is introduced. Accordingly, this provides an idea about the deviation of the risk inherent to nationality with respect to the average of exogenous risks. The figures below depict the case of risk aversion

($\alpha = 40$) and extreme risk aversion – in both figures, the two smoothed curves lying at the bottom are those of the order 3. At the order 2 ($\nu = 2$), it seems that the health matrix with the nationality risk factor is the most unequal. However this result is ambiguous since the curves cross for a risk aversion of $\alpha = 40$ but not for extreme risk aversion. The order 3 ($\nu = 3$) asserts that the achievement curve associated with the average risk factor dominates the nationality one, in such a way that more socio-economic health inequality is recorded with the nationality risk factor.

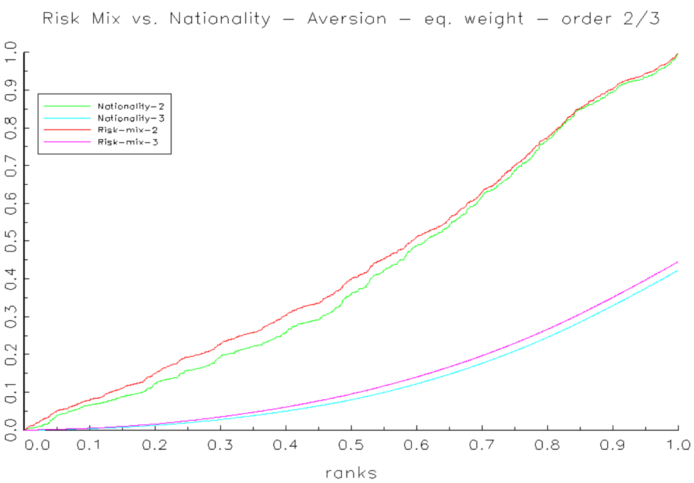

Figure 5a: Risk aversion, $\alpha = 40$

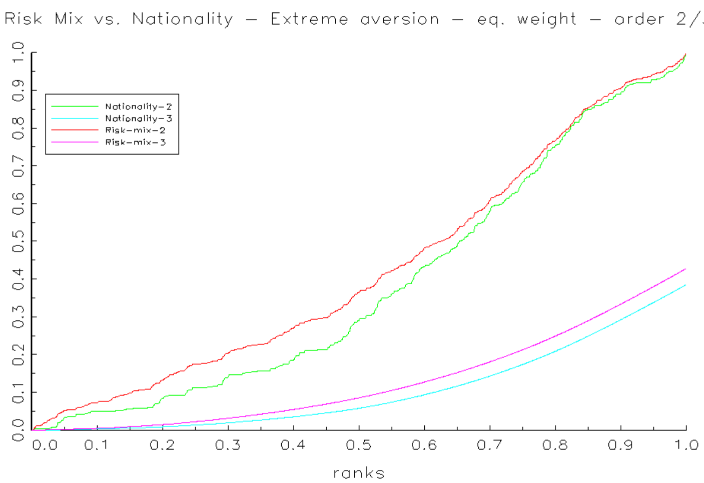

Figure 5b: Extreme risk aversion, $\alpha = \infty$

6 Robustness check

6.1 Weight vector

One principal debate in the literature on multidimensional poverty is the role and the intensity of the weight attached to each dimension. Thus, the selection of the weight vector Θ employed for the aggregation of the dimensions of health in our synthetic indicators is very important.

As we have already explained, in our case, we use the equal weighting scheme in order to give the same importance to each single health dimension. However, other systems of weights take into account the intensity of the dimensions among the population (Cerioli and Zani, 1990) while others limit the influence of those dimensions that are highly correlated (Betti and Verma, 1998). Specifically, Cerioli and Zani (1990) give a stronger weight to relatively rare dimensions:

$$\theta_k^{CZ} = \log\left(\frac{1}{\bar{h}_k}\right),$$

where $\bar{h}_k$ is the arithmetic mean of the k-th health dimension (that is of the $\iota(h_k(p))$ over all p). The weight of any dimension of health proposed by Betti and Verma (1998) is defined as follows:

$$\theta_k^{BV} = \theta_k^a \cdot \theta_k^b,$$

where θ_k^a only depends on the distribution of the k-th dimension, whereas θ_k^b depends on the correlation between k and the other dimensions. In particular, θ_k^a is determined by the coefficient of variation of $\iota(h_i(p))$,

$$\theta_k^a = \frac{n \int_0^1 [\iota(h_k(p)) - (\bar{h}_k)^2] dp}{(n\bar{h}_k)^{\frac{1}{2}}}.$$

The weights θ_k^b are computed as follows:

$$\theta_j^b = \left[1 + \sum_{k=1}^{K} \rho_{k,k'} F(\rho_{k,k'} < \rho_H)\right]^{-1} \left[1 + \sum_{k=1}^{K} \rho_{k,k'} F(\rho_{k,k'} \geq \rho_H)\right]^{-1},$$

where $\rho_{k,k'}$ is Pearson's correlation coefficient between dimensions k and k' and $F(\cdot)$ is an indicator function valued to be 1 if the expression in brackets is true and 0 otherwise. The parameter ρ_H is a pre-determined cut-off correlation level between the two dimensions.[10] In other words, it separates high and low correlations. The term θ_k^b is the inverse of a measure of average correlation of dimension k with the others. The larger the average correlation with dimension k, the lower the resulting weight for that dimension.

As depicted in the figures below, we have computed the achievement curves using the three proposed weighting schemes: weight 1 is the equal weight (EW), weight 2 is Cerioli and Zani's weight (CZ), and weight 3 is Betti and Verma's weight (BV). In our approach, it is possible to see that the achievement curves are not sensitive to the three possible normalized weights Θ. It is noteworthy that, in the extreme aversion case, the weight vector is totally independent of the dimensions, so that the achievement curves are exactly the same for any given weight vector.

[10]Betti and Verma (1998) suggest setting ρ_H so as to divide the ordered set of correlations at the point of the largest gap.

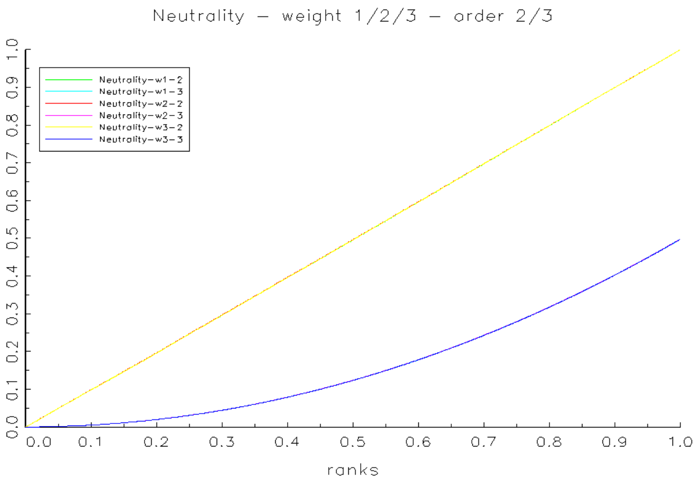

Figure 6a: Risk neutrality

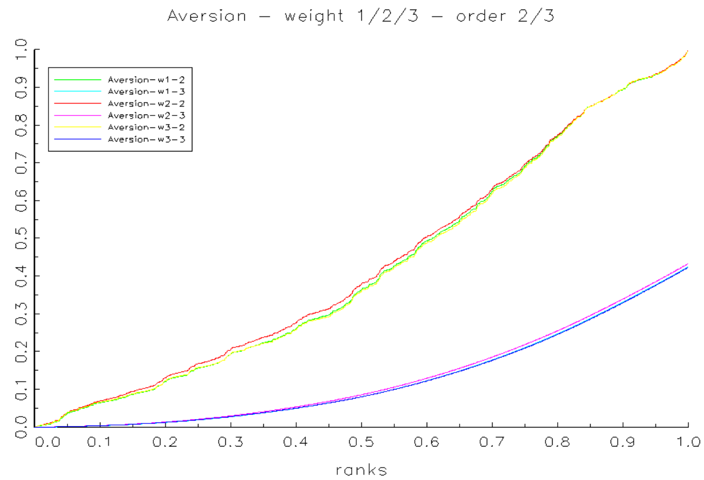

Figure 6b: Risk aversion $\alpha = 40$

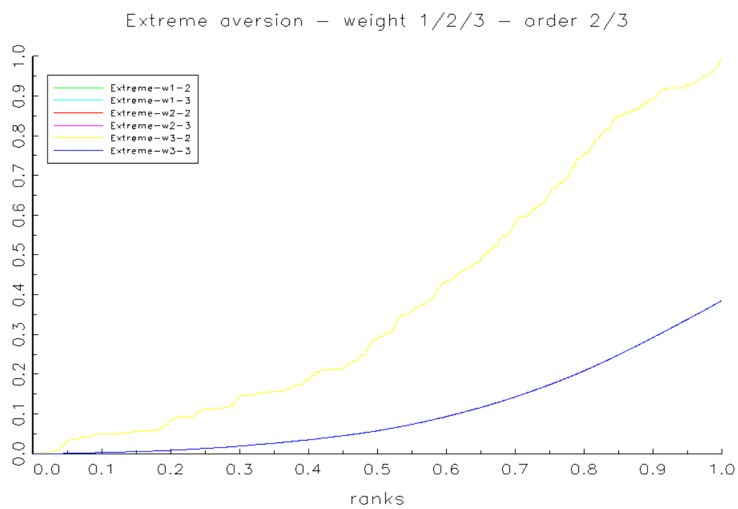

Figure 6c: Extreme risk aversion

6.2 Averaging the risk factors

In order to aggregate the risk factors into one single variable in the case of aversion risk (Eq. (2.5)), it is possible to alternatively use the EW, CZ and BV weighting schemes. Since the risk factors are constructed as Boolean variables, the variations of the achievement curves are very low, suggesting that the way of averaging has no impact on their curvature. As we can see for the aversion case ($\alpha = 40$), the achievement curves are almost the same (and exactly the same for the neutrality and extreme cases).

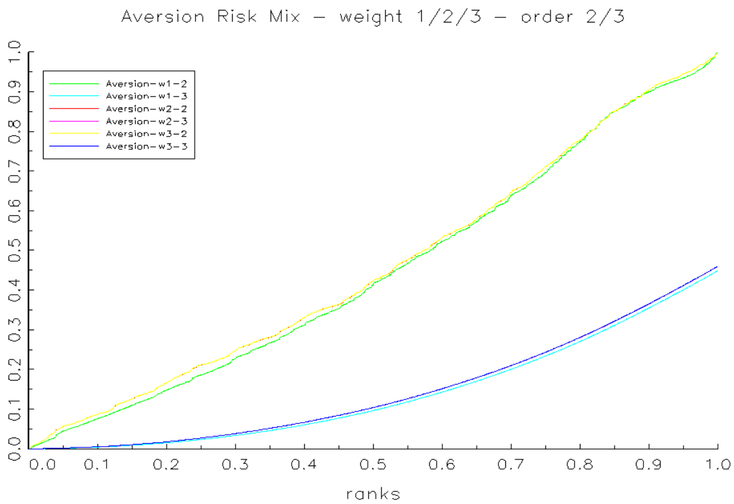

Figure 7a: Risk aversion $\alpha = 40$

7 Conclusion

The two-parameter family of socio-economic health inequality measures $\{I\}_{\nu,\alpha}$ allows risk and inequality to be captured in order to provide a non-ambiguous ranking of multidimensional health distributions. In other words, the social planner behind the veil of ignorance is endowed with a bidimensional view, which is necessary to apprehend the impact of risk factors (that agents do not control for) on the level of inequality in a society.

Further investigations may be done in this way, such as dealing with multiple risk factors, for instance. This could determine some priority in the redistribution to be made to the non healthy individuals affected by exogenous risk factors.

Finally, some applications could be done in order to compare the nature of different risk factors. Although, our study lays the emphasis on risks being circumstances, it would be informative to pay attention to risk under control. This would produce an evaluation and a comparison of the impact of these different risks on the overall level of inequality in the society.

Appendix A1

Proof of Theorem 4.1.

Proof:

Sufficiency.

Note that, for all $\alpha > 0$ and $\nu \in \{1, 2, 3, \ldots\}$, for all inequality indices $I_{\nu,\alpha}$, we get,

$$I_{\nu,\alpha}(\mathbf{H})-I_{\nu,\alpha}\left(\mathbf{G}\right)=-\int_0^1 v(p)\left[\left(1-\|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta\right)-\left(1-\|\Upsilon(\mathbf{G}(p))\|_\alpha^\Theta\right)\right]dp.$$

Thus, by Definition 4.1, $I_{\nu,\alpha}(\mathbf{H})-I_{\nu,\alpha}\left(\mathbf{G}\right)\geq 0$ is equivalent to:

$$\int_0^1 v(p)\left[A_{\mathbf{G}}^{1,\alpha}(p)-A_{\mathbf{H}}^{1,\alpha}(p)\right]dp\geq 0.$$

Integrating by parts $\int_0^1 v(p)A_{\mathbf{H}}^{1,\alpha}(p)dp$, for all $\nu\in\{1,2\ldots\}$, entails:

$$\int_0^1 v(p)A_{\mathbf{H}}^{1,\alpha}(p)dp=\left|v(p)A_{\mathbf{G}}^{2,\alpha}(0)\right|_0^1-\int_0^1 v^{(1)}(p)A_{\mathbf{H}}^{2,\alpha}(p)dp.$$

Since $v^{(\nu)}(1)=0$ and by definition $A_{\mathbf{G}}^{1,\alpha}(0)=0$, then:

$$\int_0^1 v(p)A_{\mathbf{H}}^{1,\alpha}(p)dp=-\int_0^1 v^{(1)}(p)A_{\mathbf{H}}^{2,\alpha}(p)dp.$$

Integrating the previous relation $\nu-1$ times provides:

$$\int_0^1 v(p)A_{\mathbf{H}}^{\nu,\alpha}(p)dp=(-1)^{\nu-1}\int_0^1 v^{(\nu-1)}(p)A_{\mathbf{H}}^{\nu,\alpha}(p)dp.$$

Hence, for all $\alpha>0$:

$$I_{\nu,\alpha}(\mathbf{H})-I_{\nu,\alpha}\left(\mathbf{G}\right)=(-1)^{(\nu-1)}\int_0^1 v(p)^{(\nu-1)}\left(p\right)\left[A_{\mathbf{G}}^{\nu,\alpha}\left(p\right)-A_{\mathbf{H}}^{\nu,\alpha}\left(p\right)\right]. \quad (7.1)$$

Note that $(-1)^{(\nu-1)}\int_0^1 v(p)^{(\nu-1)}dp\geq 0$. If $A_{\mathbf{G}}^{\nu,\alpha}\left(p\right)-A_{\mathbf{H}}^{\nu,\alpha}\left(p\right)\geq 0$ for all $p\in[0,1]$, then it results that $I_{\nu,\alpha}(\mathbf{H})-I_{\nu,\alpha}\left(\mathbf{G}\right)\geq 0$.
Necessity.
Consider the following weight function:

$$v^{(\nu-2)}\left(p\right)=\begin{cases}(-1)^{\nu-2}\epsilon & p\leq\overline{p}\\ (-1)^{\nu-2}\left(\overline{p}+\epsilon-p\right) & \overline{p}<p\leq\overline{p}+\epsilon\\ 0 & p>\overline{p}+\epsilon\end{cases}.$$

It follows that,

$$v^{(\nu-1)}\left(p\right)=\begin{cases}0 & p\leq\overline{p}\\ (-1)^{\nu-1} & \overline{p}<p\leq\overline{p}+\epsilon\\ 0 & p>\overline{p}+\epsilon\end{cases}. \quad (7.2)$$

Assume that $A_{\mathbf{G}}^{\nu,\alpha}\left(p\right)-A_{\mathbf{H}}^{\nu,\alpha}\left(p\right)<0$ on an interval $[\overline{p},\overline{p}+\epsilon]$ for some ϵ close to 0. Substituting (7.2) in (7.1) yields $I_{\nu,\alpha}(\mathbf{H})-I_{\nu,\alpha}\left(\mathbf{G}\right)<0$, a contradiction. ■

Appendix A2

Table A.2.I: Depression

Depression scale Euro-d*	Degree of membership
Non depressed (0 dimension)	0
Between 1 and 11 dimensions	$1-(12-X_i)/12$
Completely depressed (12 dimensions)	1

*This composite indicator takes into consideration the following dimensions: depression, pessimism, suicidal thought, guilty, sleep, interest, irritability, appetite, tiredness, concentration, enjoyment, tearfulness.

Table A.2.II: Memory

Memory and ability to think about things		Degree of membership
Four questions have been asked*	Knows all	0
	Knows 3 of 4	0.3
	Knows 2 of 4	0.6
	Knows 1 of 4	0.9
	Doesn't know	1

*Which day of the month is it? Which month is it? Which year is it? Can you tell me which day of the week it is?

Table A.2.III: Chronic illness

Long term illness		Degree of membership
Do you have any long-term health problems, illness, disability or infirmity?	No	0
	Yes	1

Table A.2.IV: Other illnesses

Other illnesses		Degree of membership
Did a doctor say you have any of the following conditions?*	No	0
	One of these conditions	0.75
	Two or more of these conditions	1

*A heart problem; High blood pressure or hypertension; High blood cholesterol; A stroke or cerebral vascular disease; Diabetes or high blood sugar; Chronic lung disease such as chronic bronchitis or emphysema; Asthma; Arthritis, including osteoarthritis, or rheumatism; Osteoporosis; Cancer or malignant tumor; Stomach or duodenal ulcer; Parkinson disease; Cataracts; Hip fracture or femoral fracture.

Table A.2.V: Limitation activities 1

Health and activities		Degree of membership
Because of a health problem, do you have difficulty doing any of the following activities?*	No	0
	One of these activities	0.15
	Two of these activities	0.25
	Three of these activities	0.50
	Four of these activities	0.75
	Five or more of these activities	1

*Walking 100 meters; Sitting for about two hours; Getting up from a chair after sitting for long periods; Climbing several flights of stairs without resting; Climbing one flight of stairs without resting; Stooping, kneeling or crouching; Reaching or extending your arms above shoulder level; Pulling or pushing large objects like a living room chair; Lifting or carrying weight over 5 kilos, like a heavy bag of groceries.

Table A.2.VI: Limitation activities 2

Health and activities		Degree of membership
Because of a health problem, do you have difficulty doing any of the following activities?*	No	0
	One of these activities	0.15
	Two of these activities	0.25
	Three of these activities	0.50
	Four of these activities	0.75
	Five or more of these activities	1

*Dressing, including putting on shoes and socks; Walking across a room; Bathing or showering; Eating, such as cutting up for your food; Getting in or out of bed; Using the toilet, including getting up or down; Using a map to figure out how to get around in a strange place; Preparing a hot meal; Shopping for groceries; Making telephone calls; Taking medications; Doing work around the house or garden; Managing money, such as paying bills and keeping track or expenses.

Table A.2.VII: Weight problems[11]

Weight problems $\leq$ 65 years old	**Degree of membership**
IMC $<$ 17.5	1
$17.5 \leq$ IMC $<$ 18.5	(18.5 − IMC) / (18.5 − 17.5)
$18.5 \leq$ IMC $<$ 25	0
$25 \leq$ IMC $<$ 30	(30 − IMC) / (30 − 25)
IMC $\geq$ 30	1
Weight problems $\geq$ 66 years old	**Degree of membership**
IMC$<$ 21	1
$21 \leq$ IMC $<$ 23	(23 − IMC) / (23 − 21)
$23 \leq$ IMC $<$ 27	0
$27 \leq$ IMC $<$ 30	(30 − IMC) / (30 − 27)
IMC $\geq$ 30	1

Table A.2.VIII: Eyesight

Eyesight distance and reading*	**Degree of membership**
Both are E or VG	0
One is E or VG, the other is G or F	0.15
One is E or VG, the other is P	0.25
Both are G or F	0.30
One is G or F, the other is P	0.60
Both are P	1

*E: excellent; VG: very good; G: good; F: fair; P: poor

[11]The Body Mass Index is calculated as follows: IMC= weight (in kg)/height^2 (in meters).

Table A.2.IX: Hearing

Hearing		Degree of membership
Is your hearing*	Excellent or Very good	0
	Good or Fair	0.15
	Poor	1

*We have also considered individuals who are using a hearing aid as usual.

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